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A Truly Crazy Idea About Type-IIB Supergravity and Heterotic Sigma-Models [1], [2]

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ABSTRACT

We construct an explicit and manifestly (1,0) heterotic sigma-model where the background fields are those of 10D, N = IIB supergravity.

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I. Introduction

Some time ago, we began to notice that heterotic sigma models have a great deal more flexibility than originally thought. Our first major success along these lines was the construction of an explicit manifestly (1,0) supersymmetric nonlinear sigma-model to describe the 4D, N = 4, SO(44) supergravity-Yang-Mills system coupled to a heterotic string [1]. Later we were able to construct such a model to describe the explicit coupling of the complete 496 $E_8 \otimes E_8$ gauge bosons of the 10D heterotic string [2],[3]. Finally, we have also proposed that it is actually possible to couple the complete spectrum of 4D, N = 8 supergravity to the heterotic string [4]! This last proposal is by no means a proof that the heterotic string has a completely consistent formulation whose massless sector describes 4D, N = 8 supergravity. But it is certainly a suggestive observation that a careful study of this question needs to be undertaken.

II. A Brief Review of Supergravity-Heterotic Sigma-Models

The elements for constructing heterotic sigma-models is by now very well known. (For general review see [5].) In this section, we give a brief review. The basic structure of a (1,0) heterotic σ-model [6] contains the NS-NS fields ($g_{\underline{mn}}$, $b_{\underline{mn}}$ and Φ) that appear in the standard σ-model [7], axion coupling [8]

$$S_\sigma + S_{WZNW} ~=~ \frac{1}{2\pi\alpha'}\int d^2\sigma d\zeta^- E^{-1}[i\tfrac{1}{2}(g_{\underline{mn}}(X) + b_{\underline{mn}}(X))\,(\nabla_+ \mathbf{X}^{\underline{m}})(\nabla_= \mathbf{X}^{\underline{n}})~] ~~, \quad (1)$$

and dilaton coupling terms [9]

$$S_{FT} ~=~ \int d^2\sigma d\zeta^- E^{-1}\Phi(X)\Sigma^+ \quad . \quad (2)$$

(For our notational conventions see reference [1].) It is often convenient to add the three action above together to form

$$S_{NS} = \int d^2\sigma d\zeta^- E^{-1}[i\tfrac{1}{2}(\eta_{\underline{ab}} + B_{\underline{ab}}(X))\Pi_+{}^{\underline{a}}\Pi_={}^{\underline{b}} ~+~ \Phi(X)\Sigma^+ ~] ~~, \quad (3)$$

where $\Pi_+{}^{\underline{a}} \equiv (1/\sqrt{2\pi\alpha'})(\nabla_+ \mathbf{X}^{\underline{m}}) e_{\underline{m}}{}^{\underline{a}}$ and $\Pi_={}^{\underline{a}} \equiv (1/\sqrt{2\pi\alpha'})(\nabla_= \mathbf{X}^{\underline{m}}) e_{\underline{m}}{}^{\underline{a}}$ Finally, it was shown how the fermionic formulation of the original heterotic string action [10] could be generalized [11]

$$S_R ~=~ \int d^2\sigma d\zeta^- E^{-1}[~-\tfrac{1}{2}(~\eta_-{}^{\hat{I}}\nabla_+\eta_-{}^{\hat{I}} ~+~ \Pi_+{}^{\underline{a}}\eta_-{}^{\hat{I}} A_{\underline{a}\hat{I}\hat{J}}(X)\eta_-{}^{\hat{J}})~] ~~, \quad (4)$$

to at least include the fields of the SO(32) version. All of the above expressions are in terms of (1,0) superfields. The final two major advances in the heterotic sigma-model

description of the massless sector of the heterotic string occurred when it became possible to give an explicit Lagrangian that realized the non-abelian chiral (1,0) superfield bosonization of S_R [1] and later this result was generalized in a manner that was consistent with manifest (1,0) superfield compactification [2],[3]. The interested reader should refer to these last two references for details regarding the non-abelian Lefton-Righton Thirring Models (LRTM) that accomplished the final two advances.

A simpler version of the LRTM theories arises when only abelian groups are considered. It was within the confines of this class of models that we found our surprising result [4] that a special form of 4D, N = 8 supergravity could actually be coupled to a heterotic σ-model! This special form of 4D, N = 8 supergravity that we call $Spin(6)$, N = 8 supergravity contains in addition to the NS fields, twenty-eight spin-1 fields ($\widetilde{A}_{\underline{a}}^{[ij]}$, $A_{\underline{a}}$, $A_{\underline{a}}^{[i'j']}$, and $A_{\underline{a}}^{[i'j'][k'l']}$) and sixty-eight scalar fields ($\Phi_{[ij]}$, $\Phi_{[ij][i'j']}$, $\Phi_{[p'q'][i'j'][k'l']}$, and $\widetilde{\Phi}_{[k'l']}$). We collectively may refer to these as Ramond sector fields. The coupling of these to the heterotic string is accomplished by first introducing lefton and righton superfields ${\varphi_L}^{\hat{\alpha}}$ and ${\varphi_R}^{\hat{I}}$ on the world sheet and then replacing the action S_R by

$$
\begin{aligned}
S_{R2} = \int d^2\sigma d\zeta^- E^{-1} i\tfrac{1}{2}[\ & (L_{=}^{\hat{\alpha}} + \Gamma_{=}^{\hat{\alpha}})(L_{+}^{\hat{\alpha}} - {\Lambda_+}^{=}(L_{=}^{\hat{\alpha}} + \Gamma_{=}^{\hat{\alpha}})) + L_{+}^{\hat{\alpha}}\Gamma_{=}^{\hat{\alpha}} \\
& +(R_{+}^{\hat{I}} + 2\Gamma_{+}^{\hat{I}})R_{=}^{\hat{I}} - i{\Lambda_=}^{++}(R_{+}^{\hat{I}} + \Gamma_{+}^{\hat{I}})\nabla_+(R_{+}^{\hat{I}} + \Gamma_{+}^{\hat{I}}) \\
& +4S^{\alpha\hat{I}}\mathrm{R}_{=}^{\hat{I}}\mathrm{L}_{+}^{\hat{\alpha}} - \ 4{\Lambda_+}^{=}(M^{-1})^{\hat{I}\hat{K}}S^{\hat{\alpha}\hat{I}}S^{\hat{\alpha}\hat{J}}\Sigma_{=}^{\hat{J}}\Sigma_{=}^{\hat{K}} \\
& -4i{\Lambda_=}^{\ddagger}S^{\hat{\alpha}\hat{I}}\mathrm{L}_{+}^{\hat{\alpha}}\nabla_+(S^{\hat{\beta}\hat{I}}\mathrm{L}_{+}^{\hat{\beta}})\ ] \quad ,
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
\mathrm{L}_{+}^{\hat{\alpha}} &= L_{+}^{\hat{\alpha}} - {\Lambda_+}^{=}(L_{=}^{\hat{\alpha}} + \Gamma_{=}^{\hat{\alpha}}) \quad , \quad L_{A}^{\hat{\alpha}} \equiv \nabla_A{\varphi_L}^{\hat{\alpha}} \quad , \quad R_{A}^{\hat{I}} \equiv \nabla_A{\varphi_R}^{\hat{I}} \quad , \\
\mathrm{R}_{=}^{\hat{I}} &= R_{=}^{\hat{I}} - i[{\Lambda_=}^{\ddagger}\nabla_+(R_{+}^{\hat{I}} + \Gamma_{+}^{\hat{I}}) + \tfrac{1}{2}(\nabla_+{\Lambda_=}^{\ddagger})(R_{+}^{\hat{I}} + \Gamma_{+}^{\hat{I}})] \quad , \\
\Sigma_{=}^{\hat{I}} &= \mathrm{R}_{=}^{\hat{I}} - 2i[{\Lambda_=}^{\ddagger}\nabla_+(S^{\hat{\beta}\hat{I}}\mathrm{L}_{+}^{\hat{\beta}}) + \tfrac{1}{2}(\nabla_+{\Lambda_=}^{\ddagger})S^{\hat{\beta}\hat{I}}\mathrm{L}_{+}^{\hat{\beta}}] \quad , \\
(M)^{\hat{I}\hat{J}} &= \delta^{\hat{I}\hat{J}} - 4i(\nabla_+{\Lambda_+}^{=}){\Lambda_=}^{\ddagger}S^{\hat{\alpha}\hat{I}}S^{\hat{\alpha}\hat{J}} \quad .
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
{\Gamma_=}^{\hat{\alpha}} &\equiv {\Pi_=}^{\underline{a}}{A_{\underline{a}}}^{\hat{\alpha}}(X) \quad , \quad {A_{\underline{a}}}^{\hat{\alpha}} = (\widetilde{A}_{\underline{a}}^{[ij]}) \quad , \\
{\Gamma_+}^{\hat{I}} &\equiv {\Pi_+}^{\underline{a}}{A_{\underline{a}}}^{\hat{I}}(X) \quad , \quad {A_{\underline{a}}}^{\hat{I}} = (A_{\underline{a}}, A_{\underline{a}}^{[i'j']}, A_{\underline{a}}^{[i'j'][k'l']}) \quad .
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
\Phi_{\hat{\alpha}\hat{I}} &\equiv \ (\ \Phi_{[ij]},\ \Phi_{[ij][i'j']},\ \Phi_{[p'q'][i'j'][k'l']},\ \delta_{i'[i}\delta_{j]j'}\widetilde{\Phi}_{[k'l']} - \delta_{k'[i}\delta_{j]l'}\widetilde{\Phi}_{[i'j']}\ ) \quad , \\
S_{\hat{\alpha}\hat{I}}(X) &\equiv \Phi_{\hat{\alpha}\hat{I}}(X) \quad .
\end{aligned} \tag{8}
$$

The world sheet lefton and righton superfields ${\varphi_L}^{\hat{\alpha}}$ and ${\varphi_R}^{\hat{I}}$ parametrize the Lie algebra $\mathrm{U}_L(1)^6 \otimes \mathrm{U}_R(1)^{22}$. The spin-1 spacetime gauge fields precisely gauge these world sheet currents.

III. Coupling the Type-IIB Supergravity Background to a (1,0) Heterotic σ-model

At first glance, what we are proposing to do may seem completely unreasonable to the knowledgeable reader. The standard interpretation of the heterotic string is that it describes at most 10D, N = 1 supergravity coupled to 10D, N = 1 Yang-Mills theory in the massless sector of the string. We have argued for a long time that as far as the heterotic sigma-models go, a fiber bundle interpretation is quite natural [1]. As such, what we undertake below is just the change of the fiber that is taken as the input to the 10D supergravity-heterotic σ-model.

The complete spectrum of the Type-IIB supergravity theory is well known, it consists of the fields of 10D, N = 1 supergravity (${e_{\underline{a}}}^{\underline{m}}$, ${\psi_{\underline{a}}}^{\alpha}$, $B_{\underline{ab}}$, χ_α, Φ) added to a multiplet with the spectrum (${\psi'_a}^{\alpha}$, ${\mathcal{F}_\alpha}^{\beta}$, ${\chi'}_\alpha$). Here ${\mathcal{F}_\alpha}^{\beta}$ denotes a Duffin-Kemmer-Petiau (DKP) field whose explicit form is given by,

$$ {\mathcal{F}_\alpha}^{\beta} \equiv \mathrm{A}{\delta_\alpha}^{\beta} \;+\; \tfrac{1}{2}\mathrm{A}_{\underline{ab}}{(\sigma^{\underline{ab}})_\alpha}^{\beta} \;+\; \tfrac{1}{24}\mathrm{A}_{\underline{abcd}}{(\sigma^{\underline{abcd}})_\alpha}^{\beta} \tag{9} $$

The fact that the bosonic fields above fit so nicely into our 10D sigma-matrix representations will be used to our advantage in the following. The bosonic spectrum of the Type-IIB supergravity theory is thus given by (${e_{\underline{a}}}^{m}$, $B_{\underline{ab}}$, Φ; ${\mathcal{F}_\alpha}^{\beta}$).

Now let us switch to the 2D world-sheet of the heterotic string. We can define a DKP field on the world sheet by

$$ {\phi_\alpha}^{\beta}(\tau,\sigma) \;\equiv\; \phi(\tau,\sigma){\delta_\alpha}^{\beta} \;+\; \tfrac{1}{2}\phi_{\underline{ab}}(\tau,\sigma){(\sigma^{\underline{ab}})_\alpha}^{\beta} \;+\; \tfrac{1}{24}\phi_{\underline{abcd}}(\tau,\sigma){(\sigma^{\underline{abcd}})_\alpha}^{\beta} \quad . \tag{10} $$

One of the interesting features of the DKP fields defined as above is that they form a closed algebra under ordinary commutation

$$ [\ {(\phi_1)_\alpha}^{\beta}\ ,\ {(\phi_2)_\beta}^{\gamma}\ ] \;=\; {(\phi_3)_\alpha}^{\gamma} \quad , \tag{11} $$

where

$$ {(\phi_3)_\alpha}^{\beta} \;=\; \tfrac{1}{2}(\phi_3)_{\underline{lm}}(\tau,\sigma){(\sigma^{\underline{lm}})_\alpha}^{\beta} \;+\; \tfrac{1}{24}(\phi_3)_{\underline{lmnp}}(\tau,\sigma){(\sigma^{\underline{lmnp}})_\alpha}^{\beta} \quad . \tag{12} $$

In the above the fields are defined as,

$$ \begin{aligned} (\phi_3)_{\underline{lm}}(\tau,\sigma) \equiv\; & \tfrac{1}{2}\ [\ 4(\phi_1)_{\underline{ab}}\ (\phi_2)_{\underline{ef}}\ \eta^{\underline{bf}}\ {\delta_{[\underline{m}}}^{\underline{a}}{\delta_{\underline{l}]}}^{\underline{e}} \\ & +\ \tfrac{2}{3}\ (\phi_1)_{\underline{abcd}}\ (\phi_2)_{\underline{efgh}}\ \eta^{\underline{bh}}\eta^{\underline{cg}}\eta^{\underline{df}}\ {\delta_{[\underline{m}}}^{\underline{a}}{\delta_{\underline{l}]}}^{\underline{e}}\ ] \quad , \end{aligned} \tag{13} $$

and

$$(\phi_3)_{\underline{lmnp}}(\tau,\sigma) \equiv \; -\tfrac{1}{3}\, [\; (\phi_1)_{\underline{ab}}\; (\phi_2)_{\underline{efgh}}\; \eta^{\underline{b}\;\underline{unh}}\; \delta_{[\underline{l}}{}^{\underline{a}}\; \delta_{\underline{m}}{}^{\underline{e}}\; \delta_{\underline{n}}{}^{\underline{f}}\; \delta_{\underline{p}]}{}^{\underline{g}}$$

$$+\;\; (\phi_1)_{\underline{abcd}}\; (\phi_2)_{\underline{ef}}\; \eta^{\underline{df}}\; \delta_{[\underline{l}}{}^{\underline{a}}\; \delta_{\underline{m}}{}^{\underline{b}}\; \delta_{\underline{n}}{}^{\underline{c}}\; \delta_{\underline{p}]}{}^{\underline{e}}\;] \qquad (14)$$

$$-\; 2\; (\phi_1)_{\underline{abcd}}\; (\phi_2)_{\underline{efgh}}\; \eta^{\underline{dh}}\; \epsilon_{\underline{abcefglmnp}} \quad .$$

Closure of the algebra of such fields is precisely what is needed to be able form a group by exponentiation. We can define group elements by $[U]_\alpha{}^\beta \equiv [\exp(\phi)\,]_\alpha{}^\beta$. This group with 256 Lie-algebra generators represented by the σ-matrices above is non-compact. Since not very much is known about the Kac-Moody extension of such quantities, the appearance of this σ-model construction is very suggestive toward the possibility of new complete heterotic strings based on such non-compact groups.

It is now our proposal to take this non-compact Lie group and use it in place of the standard $\mathrm{E}_8 \otimes \mathrm{E}_8$, $SO(32)$ or $SO(16) \otimes SO(16)$ of the known consistent heterotic string theories. At the level of the σ-model this is simple.

$$\begin{aligned} S'{}_R \;=\; -\frac{1}{2\pi}\int d^2\sigma d\zeta^- E^{-1} i\tfrac{1}{2} Tr\{\; &(R_+ \;+\; 2\Gamma_+)R_= \\ &+ i\Lambda_={}^{+\!\!\!+}(R_+ \;+\; \Gamma_+)\nabla_+(R_+ \;+\; \Gamma_+) \\ &+ \frac{2}{3}\Lambda_={}^{+\!\!\!+}\{\,(R_+ \;+\; \Gamma_+)\,,\; (R_+ \;+\; \Gamma_+)\,\}\; (R_+ \;-\; \tfrac{1}{2}\Gamma_+) \\ &+ \int_0^1 d\,y\; [\,(\frac{d\widetilde{U}}{dy}\widetilde{U}^{-1}\,)[\,\nabla_=((\nabla_+\widetilde{U})\widetilde{U}^{-1}) \;-\; \nabla_+((\nabla_=\widetilde{U})\widetilde{U}^{-1})\,]\;\} \quad . \end{aligned} \qquad (15)$$

where the following definitions are used,

$$R_a \;\equiv\; U^{-1}\nabla_a U \quad , \quad \Gamma_+ \;\equiv\; \Pi_+{}^{\underline{a}} S_{\underline{a}} \quad . \qquad (16)$$

The key to actually being able to introduce the complete bosonic spectrum of 10D, N = IIB supergravity is to observe that we can define $S_{\underline{a}}$ by

$$S_{\underline{a}} \;\equiv\; (\nabla_{\underline{a}}\mathrm{A})\;\delta_\alpha{}^\beta \;+\; \tfrac{1}{2}\mathrm{F}_{\underline{abc}}\;(\sigma^{\underline{bc}})_\alpha{}^\beta \;+\; \tfrac{1}{24}\mathrm{F}^{(+)}{}_{\underline{abcde}}\;(\sigma^{\underline{bcde}})_\alpha{}^\beta \quad . \qquad (17)$$

where $\mathrm{F}_{\underline{abc}}$ is the field strength of $\mathrm{A}_{\underline{ab}}$ and $\mathrm{F}^{(+)}{}_{\underline{abcde}}$ is the self-dual part of the field strength of $\mathrm{A}_{\underline{abcd}}$. The structure above is exactly the same as that we use for the manifest realization of the standard $\mathrm{E}_8 \otimes \mathrm{E}_8$. In this more familiar case, $S_{\underline{a}}$ is replaced by $A_{\underline{a}}$, the $\mathrm{E}_8 \otimes \mathrm{E}_8$ matrix valued connection and the two dimensional DKP field is also an element of the $\mathrm{E}_8 \otimes \mathrm{E}_8$ matrix representation of the corresponding Lie group.

V. Revisiting the 4D, N = 8 Supergravity-Heterotic Sigma-Model

The supergravity heterotic sigma-model described in the last section can be reduced to any dimension less than 10 and thus provides a unifying supergravity-heterotic σ-model based viewpoint for the existence of the maximally extended Kaluza-Klein supergravity theories in all lower dimensions.

The dimensional reduction of this 10D, N = 2 supergravity-heterotic σ-model down to a 4D, N = 8 supergravity-heterotic σ-model also provides us with a second and simplified way to describe this latter model. In our previous work, we fully "split" six of the would-be 10D string coordinates into their lefton-righton components. Applying simple dimensional reduction (toroidal compactification) to the present model leads to a model wherein the six would-be 10D string coordinates are not split. This yields a simplified description of the compactified 4D, N = 8 supergravity-heterotic σ-model. This is true because the Thirring model constructed from a righton field and an ordinary field is simpler than that constructed from a righton field and a lefton field.

The reduction can be understood by looking at the following table. For simplicity we only consider the bosonic fields since those are the only ones that can appear in the supergravity-heterotic σ-models. .

D = 10, N = 2B Supergravity Reduction

D = 10	D = 4
$e_{\underline{a}}{}^{\underline{m}}$	$\begin{pmatrix} \hat{e}_{\underline{a}}{}^{\underline{m}} & A_{\underline{a}}{}^{\hat{m}} \\ 0 & \Delta_{\hat{a}}{}^{\hat{m}} \end{pmatrix}$
$G(B)_{\underline{abc}}$	$G(B)_{\underline{abc}},\ F(B)_{\underline{ab}\hat{c}},\ F(B)_{\underline{a}\hat{b}\hat{c}}$
Φ	Φ
A	A
$F(A)_{\underline{abc}}$	$F(A)_{\underline{abc}},\ F(A)_{\underline{ab}\hat{c}},\ F(A)_{\underline{a}\hat{b}\hat{c}}$
$F(A)_{\underline{abcde}}$	$F(A)_{\underline{abc}\hat{d}\hat{e}},\ F(A)_{\underline{a}\hat{b}\hat{c}\hat{d}\hat{e}}$

Table I

It is perhaps useful here to comment upon the last row of the table above. In 10D, the field $F(A)_{\underline{abcde}}$ satisfies a self-duality condition. This implies that not all of its components are independent. In the reduction to 4D, we have only retained the

independent components. Thus, the 4D, N = 8B supergravity bosonic spectrum is obtained in the form,

Spin(6), N = 8B Supergravity

Spin	4D Supergravity Field
Multiplicity	$Spin(6)$ representation
2	$e_{\underline{a}}{}^{\underline{m}}$
1	$\{1\}$
1	$\tilde{A}_{\underline{a}}{}^{\hat{m}} + F(A)_{\underline{ab}\hat{c}} + F(B)_{\underline{ab}\hat{c}} + F(A)_{\underline{ab}\hat{c}\hat{d}\hat{e}}$
28	$\{6\} + \{6\} + \{6\} + \{10\}$
0	$B_{\underline{ab}} + F(A)_{\underline{abc}}$
2	$\{1\} + \{1\}$
0	$\Phi + A + \Delta_{\hat{a}}{}^{\hat{m}} + F(A)_{\underline{a}\hat{b}\hat{c}\hat{d}\hat{e}} + F(A)_{\underline{a}\hat{b}\hat{c}} + F(B)_{\underline{a}\hat{b}\hat{c}}$
68	$\{1\} + \{1\} + \{21\} + \{15\} + \{15\} + \{15\}$

Table II

In this table, the usual axion is $B_{\underline{ab}}$. The second axion ($F(A)_{\underline{abc}}$) can be "dualized" into a scalar that becomes the 69-th such field in the model. At this point the spectra of the N = 8A and N = 8B theories coincide. It is also possible to dualize any number of the Spin(6) spin-0 multiplets replacing them by axions.

One of the most interesting aspects of such ordinary reduction of the 10D background is that T-duality must appear in the resulting supergravity-heterotic σ-model [12]. The interesting thing about this observation is that this feature seems to distinguish between 4D, N $\leq$ 4 supergravity-heterotic σ-models and 4D, N $>$ 4 supergravity-heterotic σ-models. There appears to be no obvious T-duality in the former.

VI. Summary and Conclusion

The fact that the 10D, N = IIB theory seems to fit in so naturally into a heterotic σ-model is extremely interesting and unexpected. This construction provides a natural explanation of the result in [4]. Namely that result can be simply viewed as the dimensional compactification[5] of the present 10D, N = 2B supergravity-heterotic

[5]The careful reader might object that we actually used the type-IIA theory in [4] to derive the structure of the 4D, N = 8 supergravity-heterotic σ-model theory. However, as noted there the 4D result is independent of which 10D, type-II supergravity theory is taken as the starting point.

σ-model. Stated another way, we can "oxidize" the 4D, N = 8 supergravity-heterotic σ-model upward in dimension.

On the other hand, this present result sharply raises the question as to whether there might be similar possibilities for the 10D, N = IIA theory. The complete spectrum of the Type-IIA supergravity theory consists of the fields of 10D, N = 1 supergravity ($e_{\underline{a}}{}^{\underline{m}}$, $\psi_{\underline{a}}{}^{\alpha}$, $B_{\underline{ab}}$, χ_{α}, Φ) added to a multiplet with the spectrum ($\psi_{\underline{a}\alpha}$, $\mathcal{G}_{\alpha\beta}$, χ^{α}). Here $\mathcal{G}_{\alpha\beta}$ denotes a DKP field explicitly given by,

$$\mathcal{G}_{\alpha\beta} \equiv \mathrm{A}_{\underline{a}}(\sigma^{\underline{a}})_{\alpha\beta} \; + \; \tfrac{1}{6}\mathrm{A}_{\underline{abc}}(\sigma^{\underline{abc}})_{\alpha\beta} \tag{18}$$

By analogy with the previous case, this suggest the introduction of the following world-sheet 2D DKP field

$$\phi_{\alpha\beta}(\tau,\sigma) \equiv \phi_{\underline{a}}(\tau,\sigma)(\sigma^{\underline{a}})_{\alpha\beta} \; + \; \tfrac{1}{6}\phi_{\underline{abc}}(\tau,\sigma)(\sigma^{\underline{abc}})_{\alpha\beta} \tag{19}$$

However, with a little calculation it can be seen that introduction of this alternate fiber remains problematic. It might be possible to construct some type of coset model to yield the correct structure. This is a topic for future study.

In a similar manner, one may ask about such a possibility for the 11D, N = 1 supergravity case. Here we believe that there is no hope of introducing such a theory. The main impediment here is the fact that there seems to be no way that a heterotic string can include the required eleventh zero-mode for an 11D space.

We believe that the structure proposed in this new 10D, N = IIB supergravity-heterotic σ-model (as well as that in reference [4]) is hinting at some not generally recognized new aspect of heterotic string theory. The structure we have found suggests the existence of a new 10D heterotic string whose rightons provides coordinates for a 256 dimensional non-compact Lie-algebra. Dimensional reduction of this structure would permit us to obtain almost <u>all</u> higher D <u>and</u> N-extended supergravity theories, with the exceptions of 11D, N = 1 and 10D, N = IIA theories, from heterotic models!

Added Note: Near the completion of this work, we received a preprint of recent work [13] on string theory dynamics. This work suggests another interesting explanation of the existence the class of 4D, $4 < \mathrm{N} \leq 8$ supergravity heterotic σ-models. Namely it may well be that our models are related to the newly proposed strongly coupled phases of the heterotic string.

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